WHY TEACH THIS?

Students often like to see 'tricks' or 'cheats' for calculation. It is even better if these can be used to stimulate their curiosity about why they work and provide an opportunity for algebra to reveal the answer.

ALL SQUARE

UNPICKING THE WORKINGS OF A NUMERICAL TRICK ALLOWS STUDENTS TO PUT THEIR ALGEBRA SKILLS TO USE, SAYS COLIN FOSTER

To many students, algebra is morass of letters, numbers and symbols which have to be manipulated according to arbitrary and pointless rules to obtain a 'correct answer' which has no meaning or importance outside of the question. In this lesson, algebra is used to make sense of a number trick that the teacher performs. Students are asked to explore the mechanics, experiment with some numbers, and then use algebra to try to probe exactly how and why the trick works. It's easy to check the rule on a few numerical examples by working them out, but a short piece of algebra helps students to see what is going on under the surface. This lesson provides an opportunity to work on expanding brackets and simplifying algebraic expressions, as well as making sense of the final answer in terms of the place value of the original numbers.

STARTER ACTIVITY

Q. Without using a calculator, work out these squares:
5^2 50^2 500^2 51^2 55^2 15^2

Put them in order of how difficult you think they are to do.

Give students some time to work on these, either individually or in pairs. As you circulate around the room, notice any different approaches that you see the students taking so that you can ask them to describe these in the discussion that follows.

Q. What answers did you get?

Rather than giving feedback straight away on their answers, you could just list the different answers obtained under each of the squares; for example, some possibilities are shown below (with the correct answers shown in red):

5^2	50^2	500^2	51^2	55^2	15^2
10	250	2500	251	2525	150
25	500	50 000	2501	3025	225
	2500	250 000	2601	25 250	1250

Q. Who agrees with this answer? Why? Who disagrees? How did you work it out?

Students may be surprised that 50^2 is not 10×5^2 but is actually 100×5^2, because $(10 \times 5)^2 = 10^2 \times 5^2$. They may also be surprised that 51^2 is not $50^2 + 1^2$, because $(50 + 1)^2 = 50^2 + 2 \times 50 \times 1 + 1^2$. Some might expect 50^2 to be half of 100^2, rather than a quarter of it. There is no need to try to resolve all of these difficulties now, as the main activity will give an opportunity for that. At this point it is enough if some students are surprised or confused about some of the answers.

+KEY RESOURCES

ATM has recently published Assessment in the New National Curriculum - an ATM perspective. It is a collection of tasks that illustrate how teachers can use ordinary classroom practice as the basis for assessment. The materials have been linked to the 2014 programmes of study to support busy teachers of mathematics grappling with the demise of levels. The approach taken may be applied to any task, so this resource will be valuable for those who wish to develop rigorous assessment practice that supports progress and recognises achievement in a meaningful and formative way.
www.atm.org.uk/shop/DNL120

MAIN ACTIVITY

Q. I'm going to amaze you with a trick! Look at these numbers:

15^2	25^2	35^2	45^2	55^2	65^2	75^2	85^2	95^2

Pick one of them.

Whichever one the student picks, immediately give the value, using the following trick:

1. The last two digits are always "25".

2. To get the first part, multiply the tens digit by one more than the tens digit.

So, for example, for 25 we do 2 × 3 to get the "6" and then put "25" on the end: 625.
For 75 we do 7 × 8 to get "56" and then put "25" on the end: 5625.

Students will hopefully be at least slightly impressed – they can check on their calculators that you are not bluffing! Repeat it several times.

Q. Can you see how I'm doing it?

Students might notice patterns in the answers that you provide. If not, give them some time with a calculator to see if they can work out the trick.

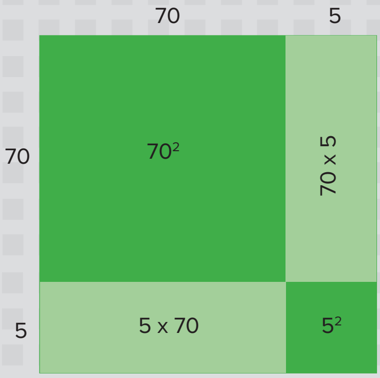
Q. Can you see the easy way of doing it? Can you work out why it works?

Encourage students to experiment and draw diagrams if that might help them to make sense of what is going on. Performing some of the calculations by hand, rather than on a calculator, may also provide insight.

DISCUSSION

You could conclude the lesson with a plenary in which the students talk about what they have worked out.

Q. Did you find any rules or patterns? Did you work out why they work?



Students might have drawn rectangle diagrams for a "generic example" like 75^2.

This might help them to see why $75^2 = 70 \times 70 + 10 \times 70 + 25 = 80 \times 70 + 25$. It is the last step here that is the difficult one!

This could be a nice opportunity for some algebra. The number "a5" ("a" tens and 5 units – you might say "a-ty five", which sounds like "eighty-five") can be written as $10a + 5$.

$(10a + 5)^2 = (10a)^2 + 2 \times (10a) \times 5 + 5^2 = 100a^2 + 100a + 25 = 100a(a + 1) + 25$.

This justifies the rule given above, because it puts "$a(a + 1)$" in the hundreds column and "25" in the units column (meaning "2" in the tens column and "5" in the units column).

Q. What will happen if we continue to larger numbers?

115^2	125^2	135^2	145^2	155^2	165^2	175^2	185^2	195^2

The answers here are:

115^2	125^2	135^2	145^2	155^2	165^2	175^2	185^2	195^2
13 225	15 625	18 225	21 025	24 025	27 225	30 625	34 225	38 025

The same pattern works here, although the product $a(a + 1)$ is generally harder to work out when a is greater than 11. However, for certain examples it is still fairly easy; for example, $195^2 = 38\,025$ just requires $19 \times 20 = 380$ and $205^2 = 42\,025$ just requires $20 \times 21 = 420$. What other "easy" ones can pupils find?

Q. Could we do something similar with numbers ending in digits other than 5?

Patterns in digits ending in 6, for example, are much less obvious:

16^2	26^2	36^2	46^2	56^2	66^2	76^2	86^2	96^2
256	676	1296	2116	3136	4356	5776	7396	9216

This is because whereas with numbers ending in 5 we had the fact that $2 \times 5 = 10$, with $2 \times 6 = 12$ it is hard to make any simplification.

INFORMATION CORNER

ABOUT OUR EXPERT

Colin Foster is an assistant professor in mathematics education in the School of Education at the University of Nottingham. He has written many books and articles for mathematics teachers (see www.foster77.co.uk).

ADDITIONAL RESOURCES

FOR OTHER INTERESTING NUMBER FACTS AND PUZZLES, SEE math.hmc.edu/funfacts

STRETCH THEM FURTHER

Another set of squares to look at is:

51^2	52^2	53^2	54^2	55^2	56^2	57^2	58^2	59^2

The answers are:

51^2	52^2	53^2	54^2	55^2	56^2	57^2	58^2	59^2
2601	2704	2809	2916	3025	3136	3249	3364	3481

and by expanding

$$(50 + a)^2 = 50^2 + 2 \times 50 \times a + a^2 = 2500 + 100a + a^2 = 100(25 + a) + a^2$$

we see that the last two digits are going to be the square of the units digit (with a leading zero where this is a single digit) and the hundreds and thousands digits are going to be 25 plus the units digit of the original number. Once you see this, with a bit of practice these are fairly easy to work out in your head.